**Enrollment No:** _______________________ **Exam Seat No:**_______________________

# C.U.SHAH UNIVERSITY
## Winter Examination-2022

**Subject Name: Group Theory**

**Subject Code: 4SC05GRT1** **Branch: B.Sc. (Mathematics)**

**Semester: 5** **Date: 23/11/2022** **Time: 02:30 To 05:30** **Marks: 70**

Instructions:
(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

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**Q-1** **Attempt the following questions:** **(14)**

**a)** In group $G$ the order of the identity element is_____. **(01)**

**b)** True or false:$(N, +)$ is group. **(01)**

**c)** Define: Order of an element of group. **(01)**

**d)** The order of 2 in $(Z_{3,}+_3)$ is ______. **(01)**

**e)** True or false:$\sigma = (1\ 2\ 3\ 4\ 5) \in S_5$ is an odd permutation. **(01)**

**f)** Define:Subgroup of group of $G$ **(01)**

**g)** State necessary and sufficient condition for non-empty subset$H$ of group$G$ to be subgroup. **(01)**

**h)** True or false:Every cyclic groupisan abelian. **(01)**

**i)** Define: Kernel ofgroup of homomorphism. **(01)**

**j)** Find $O(G)$ where $G$ =$S_4$. **(01)**

**k)** Find $< 2 >$ and $< 3 >$ in group $(Z_8, +_8)$. **(02)**

**l)** Find $\sigma\mu$ and $\sigma^2$where $\sigma = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ and $\mu = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$. **(02)**

**Attempt any four questions from Q-2 to Q-8**

**Q-2** **Attempt all questions** **(14)**

**a)** Show that $(Z_6, +_6)$ is commutative group group. **(05)**

**b)** Prove that for any group $(G,*)$ (i) the Identity element in $(G,*)$ is unique (ii) The inverse element in $(G,*)$ is unique. **(05)**

**c)** If $(G,*)$be group then prove that $(a * b * c)^{-1} = c^{-1} * b^{-1} * a^{-1}\ \forall\, a, b, c \in G$. **(04)**

**Q-3** **Attempt all questions** **(14)**

**a)** If $H_1$ and $H_2$ are two subgroups of group $G$ then prove that $H_1 \cap H_2$also subgroup of G. **(05)**

**b)** Show that $G$ is commutative group if $(ab)^n = a^n b^n n$ ; $\forall a, b \in G$ for any three **(05)**

consecutive integer $n$.

**c)** Show that the every element of finite group is of finite order. **(04)**

**Q-4** **Attempt all questions** **(14)**

**a)** For $\sigma, \mu \in S_3$ where $\sigma = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ and $\mu = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ then show that $\sigma\mu = \mu\sigma$. **(05)**

**b)** $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 3 & 1 & 5 & 6 & 4 \end{pmatrix}$ then find $\sigma^{-1}$ and $O(\sigma)$. **(05)**

**c)** If $\emptyset: G \to G^{'}$ be an isomorphism, then prove that$\emptyset(e) = e^{'}$Where$e$ and $e^{'}$ are identity element of $G$ and $G'$ respectively. **(04)**

**Q-5** **Attempt all questions** **(14)**

**a)** Let $G$ be a group and let $a \in G$ is such that $O(a) = n$ then $a^m = e$ some $m \in Z$ if and only if $n/m$. **(05)**

**b)** Show that every group of prime order is cyclic. **(05)**

**c)** Let $G$ be grou:p and for $a \neq e$ in $G$ the set $H = \{a^n | n \in Z\}$ is subgroup of $G$. **(04)**

**Q-6** **Attempt all questions** **(14)**

**a)** Show that the set $\{1, -1, i, -i\}$is cyclic group with respect to multiplication with the identity 1. **(05)**

**b)** Prove that every cyclic group is an abelian but converse is not true. **(05)**

**c)** Find the order of each element in cyclic group $(Z_6, +_6)$and also find all generators of $z_6$. **(04)**

**Q-7** **Attempt all questions** **(14)**

**a)** A subgroup $H$ of $G$ is normal subgroup of $G$ if and only if $aHa^{-1} \subset H : \forall a \in G$ **(05)**

**b)** Suppose $(G, \circ) \cong (G^{'}, *)$. Then prove that if $G$ is commutative then $G^{'}$ is commutative **(05)**

**c)** Let $G = \{1, -1, i, -i\}$ and $H = \{1, -1\}$ then show that $H$ is normal subgroup of $G$ **(04)**

**Q-8** **Attempt all questions** **(14)**

**a)** State and prove Lagrange's theorem. **(07)**

**b)** State and prove Caley's theorem **(07)**